

Meson-hyperon couplings in the bound-state approach to the Skyrme model

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Abstract

Kaon and pion coupling constants to hyperons are calculated in the bound-state approach to strangeness in the Skyrme-soliton model. The pion and kaon coupling constants are properly defined as matrix elements of source terms of the mesons sandwiched between two single-baryon states. Numerical calculation of the coupling constants shows that the bound-state approach well reproduce the empirical values.

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1 Introduction

Two methods have been proposed in order to extend the $SU(2)$ Skyrme-soliton model to strange baryons, starting with the $SU(3)_L \times SU(3)_R$ chiral symmetric meson theory. One is the $SU(3)$ collective-coordinate method, which is a natural extension of the $SU(2)$ model [1,2]. The other is the bound-state approach to strangeness, in which the flavor $SU(3)$ symmetry in the baryon sector is not presumed [3].

The nonlinear sigma model supplemented by the Wess-Zumino term is supposed to be an effective theory of QCD at the large N_c limit, where N_c is the number of colors [4]. Baryons are described in terms of solitons which are given as a classical solution of the nonlinear sigma model with appropriate stabilizing terms at leading order in the $1/N_c$ expansion. The Skyrme model has been revived as such a theory [5, 6]. Thus, extension of the $SU(2)$ Skyrme-soliton model to the strange baryons should be consistent with the $1/N_c$ expansion in the large N_c world. In this respect the bound-state approach to strangeness seems to be more suitable for the extension of the $SU(2)$ soliton model to strangeness and other heavy flavors [7,8].

Since the proposal by Callan and Klebanov [3] it has been found that the bound-state approach to strangeness works fairly well in describing the mass spectrum and magnetic moments of hyperons [9~16]. Kaon-nucleon background scattering has also been studied [17]. There have been attempts to obtain the kaon coupling constants in the bound-state approach [18,19]. However, meson-baryon scattering amplitudes including the strangeness exchange processes have not fully been developed so far within this approach.

In this paper we calculate the kaon coupling constants at hyperon-nucleon vertices and the pion ones at hyperon-hyperon vertices, where the positive-parity hyperons such as Λ , Σ and $\Sigma(1385)$ denoted as Σ^* are the P-wave bound states of antikaon to the $SU(2)$ soliton and $\Lambda(1405)$ denoted as Λ^* is the S-wave bound state. We also discuss the possible existence of negative-parity Σ states with spin 1/2 and 3/2, which are induced

from the S-wave bound kaon. The kaon and pion coupling constants are properly defined as matrix elements of the source terms of the mesons sandwiched between the two single-baryon states according to the prescription developed for resolving the Yukawa coupling problem in the $SU(2)$ Skyrme-soliton model [20,21]. Our method is simple, transparent and applicable to a wide range of meson vertices. Since we are interested in formulating the meson-baryon vertices including heavy quantum numbers, we restrict ourselves to the simplest Lagrangian which preserves the essentials of the bound-state approach.

In order to make numerical calculations we take the two sets of the parameters of the model, the pion decay constant f_π and the Skyrme constant e ; Set I consists of $f_\pi = 54\text{MeV}$ and $e = 4.84$, and Set II does of $f_\pi = 93\text{MeV}$ and $e = 4.0$. The parameters in Set I have been tuned so as to fit the masses of the nucleon and the Δ isobar by Adkins et al. [6]. In Set II the pion decay constant is kept equal to the physical value and the Skyrme constant is taken so as to give a reasonable size of the soliton and to reproduce the mass difference between the nucleon and the Δ isobar. Although the soliton has a large classical mass in Set II, it is reduced to a reasonable value if the one-loop corrections of $O(N_c^0)$ are taken into account [22,23]; for example, the resultant Skyrmion mass is 873 MeV according to ref. [23].

The sizes of the pseudovector coupling constants f_{YNK} to the positive hyperons are found to be close to those given by the compilation of the coupling constants [24,25]. The coupling constant G_{Λ^*NK} in Set II is close to the phenomenological value [26], but the one in Set I may be too large. Our results suggest that the parameter set II is more favorable than those of Set I.

In the next section the Lagrangian and Hamiltonian in the bound-state approach are given. The matrix element of the Hamiltonian in the intrinsic frame of the soliton is discussed in Appendix. The kaon and pion coupling constants to the positive parity hyperons are defined in the section 3. The kaon coupling to Λ^* is given, and the possibility

of Σ states with negative parity predicted by the model are discussed in section 4. The conclusions and discussion are given in the last section.

2 Lagrangian and Hamiltonian

For the sake of being self-contained, we give the Lagrangian and the Hamiltonian in this section. We start with the chiral $SU(3) \times SU(3)$ symmetric Lagrangian broken only by finite masses of the pion and kaon. The kaon fields are introduced as the fluctuations around the $SU(2)$ soliton according to the standard Callan-Klebanov ansatz [3]:

$$U = \sqrt{U_\pi} U_K \sqrt{U_\pi}. \quad (2.1)$$

The kaon part is written as

$$U_K = \exp \left\{ i \frac{\sqrt{2}}{f_\pi} \begin{pmatrix} 0 & K \\ K^\dagger & 0 \end{pmatrix} \right\}, \quad (2.2)$$

where

$$K = \begin{pmatrix} K^+ \\ K^0 \end{pmatrix} \quad K^\dagger = (K^-, \overline{K}^0), \quad (2.3)$$

and the pion part is

$$U_\pi = \begin{pmatrix} u & 0 \\ 0 & 1 \end{pmatrix} \quad (2.4)$$

with

$$u = \frac{1}{f_\pi} (\Phi_0 + i\tau_a \Phi_a), \quad (2.5)$$

where Φ_a is the total pion field, which consists of the classical Skyrme configuration of $O(N_c^{1/2})$ and the fluctuation field of $O(N_c^0)$ under the constraint $\Phi_0^2 = f_\pi^2 - \sum \Phi_a^2$ [20].

The Lagrangian density \mathcal{L} is written as

$$\mathcal{L} = \mathcal{L}_{sky} + \mathcal{L}_K + O(K^3), \quad (2.6)$$

where \mathcal{L}_{sky} is the $SU(2)$ Skyrme Lagrangian,

$$\mathcal{L}_{sky} = \frac{1}{2} \dot{\Phi}_a G_{ab} \dot{\Phi}_b - \mathcal{V}[\Phi_a, \nabla \Phi_a], \quad (2.7)$$

where the explicit expressions of G_{ab} and \mathcal{V} are not given here. \mathcal{L}_K is bilinear in K^\dagger and K :

$$\begin{aligned}
\mathcal{L}_K = & (D_\mu K)^\dagger (D^\mu K) - m_K^2 K^\dagger K + \frac{1}{2} m_\pi^2 \left(1 - \frac{\Phi_0}{f_\pi}\right) K^\dagger K - \frac{1}{4f_\pi^2} K^\dagger K \left\{ \partial_\mu \Phi_a X_{ab} \partial^\mu \Phi_b \right. \\
& - \frac{2}{f_\pi^2 \kappa^2} [(\partial_\mu \Phi_a X_{ab} \partial^\mu \Phi_b)(\partial_\nu \Phi_c X_{cd} \partial^\nu \Phi_d) - (\partial_\mu \Phi_a X_{ac} \partial_\nu \Phi_c)(\partial^\mu \Phi_b X_{bd} \partial^\nu \Phi_d)] \Big\} \\
& + \frac{1}{4\kappa^2 f_\pi^2} \left\{ (D_\mu K)^\dagger (D_\nu K) (\partial^\mu \Phi_a X_{ab} \partial^\nu \Phi_b) - (D_\mu K)^\dagger (D^\mu K) (\partial_\nu \Phi_a X_{ab} \partial^\nu \Phi_b) \right. \\
& - (D_\mu K)^\dagger [3i\varepsilon_{abc} \partial^\mu \Phi_a \partial^\nu \Phi_b \tau_c + 6f_\pi (V^\mu \partial^\nu \Phi_0 - V^\nu \partial^\mu \Phi_0)] (D_\nu K) \Big\} \\
& - i \frac{N_c}{4f_\pi^2} B^\mu \left\{ K^\dagger (D_\mu K) - (D_\mu K)^\dagger K \right\}, \tag{2.8}
\end{aligned}$$

where $\kappa = ef_\pi$ and

$$D_\mu = \partial_\mu + V_\mu, \tag{2.9}$$

$$V^\mu = \frac{1}{2f_\pi^2} \frac{1}{1 + \Phi_0/f_\pi} i\varepsilon_{abc} \tau_a \Phi_b \partial^\mu \Phi_c, \tag{2.10}$$

$$B^\mu = \varepsilon^{\mu\nu\alpha\beta} \frac{1}{24\pi^2} \text{Tr}(u^\dagger \partial_\nu u u^\dagger \partial_\alpha u u^\dagger \partial_\beta u) \quad \text{with} \quad \varepsilon^{0123} = -1, \tag{2.11}$$

$$X_{ab} = \delta_{ab} + \frac{\Phi_a \Phi_b}{\Phi_0^2}. \tag{2.12}$$

In order to get the Hamiltonian we extract the time-derivative terms from \mathcal{L}_K ;

$$\begin{aligned}
\mathcal{L}_K = & \dot{K}^\dagger f \dot{K} + \dot{K}^\dagger \eta_a \dot{\Phi}_a + \dot{\Phi}_a \eta_a^\dagger \dot{K} \\
& + \dot{K}^\dagger i\lambda K - K^\dagger i\lambda \dot{K} - i\Xi_a \dot{\Phi}_a \\
& + \dot{\Phi}_a \widehat{G_{ab}} \dot{\Phi}_b + \mathcal{L}', \tag{2.13}
\end{aligned}$$

where \mathcal{L}' does not include any time-derivatives of the kaon and pion fields, and

$$f = 1 + \frac{1}{4\kappa^2 f_\pi^2} (\partial_i \Phi_a X_{ab} \partial_i \Phi_b), \tag{2.14}$$

$$\lambda = \frac{N_c}{4f_\pi^2} B^0, \tag{2.15}$$

$$\begin{aligned}
\eta_a = & f V_a^0 K + \frac{1}{4\kappa^2 f_\pi^2} \left\{ -X_{ab} \partial_i \Phi_b + 3i\varepsilon_{abc} \partial_i \Phi_b \tau_c \right. \\
& \left. + 6f_\pi \left(V_a^0 \partial_i \Phi_0 + V_i \frac{\Phi_a}{\Phi_0} \right) \right\} (D_i K), \tag{2.16}
\end{aligned}$$

$$\Xi_a = 2K^\dagger (\lambda V_a^0 - \lambda_i^a V_i) K + \lambda_i^a (\partial_i K^\dagger K - K^\dagger \partial_i K). \tag{2.17}$$

When Φ_a is set to the leading classical configuration, the term $\widetilde{G_{ab}}$ bilinear in the kaon fields is of $O(N_c^{-1})$, but η_a and Ξ_a are of $O(N_c^{-1/2})$. We neglect the term $\dot{\Phi}_a \widetilde{G_{ab}} \dot{\Phi}_b$ hereafter. In the above V_a^0 and λ_i^a are defined as

$$V_a^0 \dot{\Phi}_a = V^0, \quad (2.18)$$

$$\lambda_i^a \dot{\Phi}_a = \frac{N_c}{4f_\pi^2} B_i. \quad (2.19)$$

We define the momentum fields canonically conjugate to $K(x)^\dagger$ and $\Phi_a(x)$ as

$$\Pi(x) = \frac{\delta \mathcal{L}}{\delta \dot{K}^\dagger(x)} = f \dot{K} + i\lambda K + \eta_a \dot{\Phi}_a, \quad (2.20)$$

$$\pi_a(x) = \frac{\delta \mathcal{L}}{\delta \dot{\Phi}_a(x)} = G_{ab} \dot{\Phi}_b - i\Xi_a + \dot{K}^\dagger \eta_a + \eta_a^\dagger \dot{K}. \quad (2.21)$$

Note that $\dot{\Phi}_a$, \dot{K} and \dot{K}^\dagger are solved in terms of π , Π and Π^\dagger without any constraints;

$$\dot{K} = f^{-1}[\Pi - i\lambda K - \eta_a G^{-1}_{ab} \pi_b] + O(K^3), \quad (2.22)$$

$$\dot{\Phi}_a = G^{-1}_{ab} \left\{ \pi_b + i\Xi_b - \left(\eta_b^\dagger f^{-1}[\Pi - i\lambda K - \eta_c G^{-1}_{cd} \pi_d] + h.c. \right) \right\}. \quad (2.23)$$

Then the canonical commutation relations hold among the fields;

$$[\Phi_a(\mathbf{x}, t), \pi_b(\mathbf{y}, t)] = i\delta_{ab} \delta(\mathbf{x} - \mathbf{y}), \quad (2.24)$$

$$[K_\alpha^\dagger(\mathbf{x}, t), \Pi_\beta(\mathbf{y}, t)] = i\delta_{\alpha\beta} \delta(\mathbf{x} - \mathbf{y}). \quad (2.25)$$

This is because the massive kaon fields do not contain the zero-mode wave functions of the $SU(2)$ Skyrme configuration, and because the pion fields are the total fields [20].

We can construct the Hamiltonian density through the conventional method as

$$\mathcal{H} = \mathcal{H}_{sky} + \mathcal{H}_K + \mathcal{H}_{\pi K}, \quad (2.26)$$

where \mathcal{H}_{sky} is the one of the $SU(2)$ Skyrme model

$$\mathcal{H}_{sky} = \frac{1}{2}(\pi_a G^{-1}_{ab} \pi_b) + \mathcal{V}[\phi, \nabla \phi]. \quad (2.27)$$

The Hamiltonian density \mathcal{H}_K is given as

$$\begin{aligned}\mathcal{H}_K = & [\Pi^\dagger + iK^\dagger\lambda]f^{-1}[\Pi - i\lambda K] + (D_i K)^\dagger d_{ij}(D_j K) \\ & + [m_K^2 - \frac{1}{2}m_\pi^2(1 - \frac{\Phi_0}{f_\pi}) - v_0]K^\dagger K,\end{aligned}\quad (2.28)$$

with

$$\begin{aligned}d_{ij} = & \delta_{ij}f + \frac{1}{4\kappa^2 f_\pi^2} \{-\partial_i \Phi_a X_{ab} \partial_j \Phi_b \\ & + 3i\varepsilon_{abc} \partial_i \Phi_a \partial_j \Phi_b \tau_c + 6f_\pi(V_i \partial_j \Phi_0 - V_j \partial_i \Phi_0)\},\end{aligned}\quad (2.29)$$

$$\begin{aligned}v_0 = & \frac{1}{4f_\pi^2} \left\{ \partial_i \Phi_a X_{ab} \partial_i \Phi_b + \frac{2}{\kappa^2 f_\pi^2} [(\partial_i \Phi_a X_{ab} \partial_i \Phi_b)(\partial_j \Phi_c X_{cd} \partial_j \Phi_d) \right. \\ & \left. - (\partial_i \Phi_a X_{ac} \partial_j \Phi_c)(\partial_i \Phi_b X_{bd} \partial_j \Phi_d)] \right\}.\end{aligned}\quad (2.30)$$

The last part, $\mathcal{H}_{\pi K}$, is linear in π_a and written as

$$\mathcal{H}_{\pi K} = i\pi_a G^{-1}_{ab} \Xi_b - \left\{ \left(\pi_a G^{-1}_{ab} \eta_b \right)^\dagger D^{-1} (\Pi - i\lambda K) + \text{h.c.} \right\}.\quad (2.31)$$

We should note that all the fields are defined in the laboratory system.

The Hamiltonian written in terms of the fields in the intrinsic frame of the Skyrme-soliton is given in Appendix. The bound state parameters are tabulated in Table I. The masses of the positive parity hyperons are less than the empirical values, and the mass difference between Λ and Σ is a little bit large for the both sets of the model parameters. On the other hand the mass difference between Λ^* and Λ is a little bit small. But our aim is not to search for the best parameters fitting to the baryon masses in this paper.

Table I

3 Kaon and pion couplings to positive-parity hyperons

In order to get the scattering amplitudes for the kaon, we introduce the asymptotic fields of the kaon, $K_{\text{in}}(x)$ and $K_{\text{out}}(x)$, which satisfy the free field equations:

$$K_{\alpha \text{ in}}(x) = \sum_{\mathbf{k}} \frac{1}{(2\pi)^{3/2} \sqrt{2\omega_k}} \left\{ b_{\alpha}(\mathbf{k}) e^{-ikx} + a_{\alpha}^{\dagger}(\mathbf{k}) e^{ikx} \right\}, \quad (3.1)$$

$$K_{\alpha \text{ in}}^{\dagger}(x) = \sum_{\mathbf{k}} \frac{1}{(2\pi)^{3/2} \sqrt{2\omega_k}} \left\{ a_{\alpha}(\mathbf{k}) e^{-ikx} + b_{\alpha}^{\dagger}(\mathbf{k}) e^{ikx} \right\} \quad (3.2)$$

with $\omega_k = \sqrt{\mathbf{k}^2 + m_K^2}$, and $a_{\alpha}(\mathbf{k})$ ($b_{\alpha}(\mathbf{k})$) is the annihilation operator of the antikaon (kaon) of the in-state with isospin index $\alpha = 1/2, -1/2$. The same forms are defined for out-fields. The field $K_{\alpha}(x)$ in the previous section is the interpolating field from the in-state to the out-state. The similar in- and out-fields are introduced to the pion fields, where the total field $\Phi_a(x)$ itself plays a role of the interpolating field from the in-state to the out-state [20]. The single-baryon state with the definite spin, isospin and momentum is given as the rotating and translating Skyrme-soliton including a bound-state antikaon, if the baryon has strangeness. The Fock space is spanned by the in- and out-states composed of the in- and out- creation operators of the mesons acting on the single-baryon state. The baryon state is not the eigenstate of $H = \int d^3x \mathcal{H}$, but $\langle B(\mathbf{p}') | H | B(\mathbf{p}) \rangle = E_B(\mathbf{p}) \delta(\mathbf{p}' - \mathbf{p})$ as shown in the Appendix, where $E_B = M_B + \mathbf{p}^2/2M_B$ and terms of $O(N_c^{-2})$ are discarded.

Thus, the scattering amplitude can be written through the LSZ reduction formula [27] as follows;

$$\begin{aligned} T_{\bar{K}N \rightarrow \bar{K}N} &= i(2\pi)^3 \int d^4x e^{ik'x} \langle N((\mathbf{p}') | T (J_{\beta}^{K\dagger}(x) J_{\alpha}^K(0)) \\ &\quad + \delta(x^0) [\dot{K}_{\beta}^{\dagger}(x), J_{\alpha}^K(0)] - i\omega_{k'} \delta(x^0) [K_{\beta}^{\dagger}(x), J_{\alpha}^K(0)] | N(\mathbf{p}) \rangle \end{aligned} \quad (3.3)$$

for $\bar{K}_{\alpha}(\mathbf{k}) + N(\mathbf{p}) \rightarrow \bar{K}_{\beta}(\mathbf{k}') + N(\mathbf{p}')$. The second line of the above expression consisting of the equal-time commutators is called the seagull term or the contact term. The factor $(2\pi)^3$ comes from the normalization of the baryon wave function. Strangeness exchange

scattering is described in terms of the kaon and pion source terms as follows;

$$T_{\bar{K}N \rightarrow \pi Y} = i(2\pi)^3 \int d^4x e^{iqx} \langle Y((\mathbf{p}')|T(J_b^\pi(x)J_\alpha^K(0)) \\ + \delta(x^0)[\dot{\Phi}_b(x), J_\alpha^K(0)] - i\omega_q \delta(x^0)[\Phi_b(y), J_\alpha^K(0)]|N(\mathbf{p}) \rangle \quad (3.4)$$

for $\bar{K}_\alpha(\mathbf{k}) + N(\mathbf{p}) \rightarrow \pi_b(\mathbf{q}) + Y(\mathbf{p}')$. In the above the kaon and pion source terms are defined as

$$J_\alpha^K(x) = \ddot{K}_\alpha + (-\nabla^2 + m_K^2)K_\alpha(x), \quad (3.5)$$

$$J_a^\pi(x) = \ddot{\Phi}_a + (-\nabla^2 + m_\pi^2)\Phi_a(x). \quad (3.6)$$

Inserting the single-baryon states into the time-ordered product term, we get the Born terms, the residues of which are written in terms of the kaon and pion source terms sandwiched between two single-baryon states.

3.1 Kaon couplings

The source term in this section is restricted to leading order in the $1/N_c$ expansion when it is sandwiched between the single-baryon states. The second derivative with respect to t is written as

$$\ddot{K} = f^{-1} \left\{ -2i\lambda \dot{K} + \left(v_0 + \frac{1}{2}m_\pi^2 \left(1 - \frac{\Phi_0}{f_\pi} \right) - m_K^2 \right) K + D_i(d_{ij}D_j K) \right\} \quad (3.7)$$

at $O(N_c^0)$ through the commutator $i[H_K, \dot{K}]$, where $H_K = \int d^3x \mathcal{H}_K$, and

$$\dot{K} = f^{-1}(\Pi - i\lambda K). \quad (3.8)$$

The commutator with H_{sky} gives higher order terms. Note that Eq.(3.7) is the equation of motion to K_α in the laboratory system.

When we calculate the matrix element of the source term sandwiched between the hyperon and nucleon, the kaon fields are transformed into the fields defined in the intrinsic frame, while the pion fields are reduced to the classical Skyrminion fields in the tree

approximation as follows:

$$K_\alpha(x) = A_{\alpha i} \mathcal{K}_i(\mathbf{x} - \mathbf{X}(t), t), \quad (3.9)$$

$$\Phi_a(x) = R_{ai}(t) \hat{\phi}_i(\mathbf{x} - \mathbf{X}(t)), \quad (3.10)$$

$$\Phi_0(x) = \hat{\phi}_0(\mathbf{x} - \mathbf{X}(t)), \quad (3.11)$$

$$\hat{\phi}_0(\mathbf{r}) = f_\pi \cos F(r) \quad \text{and} \quad \hat{\phi}_i(\mathbf{r}) = f_\pi \hat{r}_i \sin F(r), \quad (3.12)$$

where $\hat{\mathbf{r}} = \mathbf{r}/r$ and $F(r)$ is the profile function of the Skyrmion, $\mathbf{A}(t)$ the $SU(2)$ matrix as the collective coordinates for the iso-rotation, R_{ai} the orthogonal rotation matrix, and $\mathbf{X}(t)$ the center of the Skyrmion as another set of the collective coordinates for the translational motion.

The kaon field in the intrinsic frame is expanded as

$$\mathcal{K}(\mathbf{r}, t) = \sum_N \left\{ b_N k_N(\mathbf{r}) e^{-i\tilde{\omega}_N t} + a_N^\dagger k_N^c(\mathbf{r}) e^{i\omega_N t} \right\}, \quad (3.13)$$

where $N = \{\ell, T, T_3\}$ with ℓ being the orbital angular momentum and T and T_3 being the quantum numbers of $\mathbf{T} = \mathbf{L} + \boldsymbol{\tau}/2$, and a_N and ω_N (b_N and $\tilde{\omega}_N$) are the destruction operator and energy of the strangeness $S = -1$ (+1) kaon with the quantum number N . We only take the part associated with $S = -1$ in the eigenmode expansion, hereafter. The charge-conjugate eigenmode is written as

$$k_N^c(\mathbf{r}) = k_{\ell T}^*(r) \mathbf{Y}_{T\ell T_3}^c(\theta, \phi) \quad (3.14)$$

with

$$\mathbf{Y}_{T\ell T_3}^c = \begin{pmatrix} < T, T_3 | \ell, T_3 + \frac{1}{2}; \frac{1}{2}, -\frac{1}{2} > Y_{\ell, T_3+1/2}^*(\theta, \phi) \\ - < T, T_3 | \ell, T_3 - \frac{1}{2}; \frac{1}{2}, \frac{1}{2} > Y_{\ell, T_3-1/2}^*(\theta, \phi), \end{pmatrix} \quad (3.15)$$

where $Y_{\ell m}(\theta, \phi)$ is the usual spherical harmonics.

We now calculate the matrix elements of $J_\alpha^K(0)$, function of $\Phi(0)$ and $K(0)$, sandwiched between $< Y |$ and $| N >$:

$$< Y(\mathbf{p}') | J_\alpha^K[\Phi(0), K(0)] | N(\mathbf{p}) > = < Y(\mathbf{p}') | \mathbf{A}_{\alpha i} \mathcal{J}_i^K[\hat{\phi}(-\mathbf{X}(0)), \mathcal{K}(-\mathbf{X}(0))] | N(\mathbf{p}) >$$

$$= \frac{1}{(2\pi)^3} \int d^3r e^{i\mathbf{k}\mathbf{r}} \langle Y | \mathbf{A}_{\alpha i} \mathcal{J}_i^K [\hat{\phi}(\mathbf{r}), \mathcal{K}(\mathbf{r})] | N \rangle \equiv \frac{1}{(2\pi)^3} \langle Y | \mathbf{A}_{\alpha i} \widetilde{\mathcal{J}}_i^K(\mathbf{k}) | N \rangle \quad (3.16)$$

where $\mathbf{k} = \mathbf{p}' - \mathbf{p}$. In the above the eigenstate of $\mathbf{X}(0)$, $|\mathbf{r}\rangle$, is introduced and used is $\langle \mathbf{r} | N(\mathbf{p}) \rangle = \exp(i\mathbf{r}\mathbf{p}) / (2\pi)^{3/2} | N \rangle$ [20]. We denoted the source term in the intrinsic frame as $\mathcal{J}_i^K(\mathbf{r}, 0)$, which is written as

$$\begin{aligned} \mathcal{J}^K(\mathbf{r}, 0) &= \ddot{\mathcal{K}}(\mathbf{r}, 0) + (-\nabla^2 + m_K^2) \mathcal{K}(\mathbf{r}, 0) \\ &= \sum_N a_N^\dagger (-\omega_N^2 - \nabla^2 + m_K^2) k_N^c(\mathbf{r}) \end{aligned} \quad (3.17)$$

with ω_N being the bound state energy, where we discarded $\dot{\mathbf{A}}$ and $\ddot{\mathbf{A}}$, because they are of higher order in the $1/N_c$ expansion, and used the equation of motion to $\mathcal{K}_i(\mathbf{r})$.

For the positive parity hyperons we take values $\ell = 1$ and $T = 1/2$, and then

$$\mathcal{K}_P(\mathbf{r}, t) = k_1^*(r) e^{i\omega_1 t} \begin{pmatrix} a_{1/2}^\dagger \sqrt{\frac{2}{3}} Y_{11}^* + a_{-1/2}^\dagger \sqrt{\frac{1}{3}} Y_{10}^* \\ a_{1/2}^\dagger \sqrt{\frac{1}{3}} Y_{10}^* + a_{-1/2}^\dagger \sqrt{\frac{2}{3}} Y_{1-1}^* \end{pmatrix} \equiv k_1(r) e^{i\omega_1 t} \boldsymbol{\Omega}_1(a^\dagger; \theta, \phi), \quad (3.18)$$

where $k_1(r)$ is the radial wave function, and a_t^\dagger with $t = \pm 1/2$ denotes $a_{1,1/2,t}^\dagger$. Thus, we can write the Fourier transform of \mathcal{J}_i^K as

$$\begin{aligned} \widetilde{\mathcal{J}}^K(\mathbf{k}) &= \int d^3r e^{i\mathbf{k}\mathbf{r}} \mathcal{J}^K(\mathbf{r}) \\ &= (\omega_k^2 - \omega_1^2) \int d^3r j_1(kr) k_1(r) i \boldsymbol{\Omega}_1(a^\dagger; \hat{\mathbf{k}}), \end{aligned} \quad (3.19)$$

where ω_1 is the P-wave bound-state energy and $\hat{\mathbf{k}} = \mathbf{k}/k$.

Now, we define here the nucleon state as [6]

$$|N\rangle = |i_3, j_3\rangle = \sqrt{\frac{2}{8\pi^2}} (-1)^{1/2+i_3} D_{-i_3, j_3}^{1/2}(\Theta) |0\rangle, \quad (3.20)$$

and the hyperon Λ , Σ and Σ^* states as

$$\begin{aligned} |Y\rangle &= |I, I_3; J, J_3\rangle \\ &= \sum_t \langle J, J_3 | I, J_3 - t; \frac{1}{2}, t \rangle \sqrt{\frac{2I+1}{8\pi^2}} (-1)^{I+I_3} D_{-I_3, J_3-t}^I(\Theta) a_t^\dagger |0\rangle, \end{aligned} \quad (3.21)$$

where Θ denotes the three Euler angles of the iso-rotation [18,28]. We express the $SU(2)$ iso-rotation \mathbf{A} as $\mathbf{A}_{\alpha i} = D_{\alpha,i}^{1/2}(\Theta)$. Thus, we have

$$\langle Y | \mathbf{A}_{\alpha i} \widetilde{\mathcal{J}}_i^K(\mathbf{k}) | N \rangle = \Lambda_{YN} i(\boldsymbol{\sigma} \cdot \mathbf{k}) \widetilde{G}_1(k), \quad (3.22)$$

where the vertex function $\widetilde{G}_1(k)$ for the P-wave kaon is given as

$$\widetilde{G}_1(k) = \sqrt{4\pi} \frac{(\omega_k^2 - \omega_1^2)}{k} \int dr r^2 j_1(kr) k_1(r), \quad (3.23)$$

and $\boldsymbol{\sigma}$ should be replaced by the transition spin matrix, \mathbf{S} , from $J = 1/2$ to $J = 3/2$ for $Y = \Sigma^*$, which we define as $(S_i)_{mn} = \langle 3/2, m | 1, i; 1/2, n \rangle$. The coefficients, Λ_{YN} 's, are given in Table II, where we note that the minus sign is multiplied to the vertices with K^- meson, because the correct isospin multiplet of the atikaon is $(\overline{K}^0, -K^-)$, while our antikaon multiplet is (K^-, \overline{K}^0) .

Fixing the common mass scale at m_K for the kaon coupling constants, we define the pseudovector coupling constant f_{YNK}/m_K as

$$\frac{f_{YNK}}{m_K} = \sqrt{4\pi} \Lambda_{YN} \lim_{\omega_k \rightarrow \omega_1} \frac{\omega_k^2 - \omega_1^2}{k} \int dr r^2 j_1(kr) k_1(r), \quad (3.24)$$

because the Born term has the pole at $\omega_k = M_Y - M_N$, that is $\omega_k = \omega_1$ at leading order in the $1/N_c$ expansion. Using the asymptotic form of the normalized bound-state wave function,

$$k_1(r) \sim \alpha_1 \frac{1 + \kappa_1 r}{r^2} e^{-\kappa_1 r} \quad (3.25)$$

with $\kappa_1 = \sqrt{m_K^2 - \omega_1^2}$, we have

$$\frac{f_{YNK}}{m_K} = \sqrt{4\pi} \Lambda_{YN} \alpha_1. \quad (3.26)$$

Note that the dimension of α_1 is linear in length from the normalization condition on k_1 .

The pseudoscalar coupling constant G_{YNK} is given by

$$G_{YNK} = \frac{M_N + M_Y}{m_K} f_{YNK}. \quad (3.27)$$

According to the compilation of coupling constants of 1982-Edition [24] and Ref. [25] the empirical coupling constants are given, respectively, as

$$\begin{aligned} \left| \frac{f_{\Lambda p K^-}}{\sqrt{4\pi}} \right| &= 0.89 \pm 0.10 & \left| \frac{f_{\Sigma^0 p K^-}}{\sqrt{4\pi}} \right| &< 0.43 \pm 0.07, \\ \left| \frac{f_{\Lambda p K^-}}{\sqrt{4\pi}} \right| &= 0.94 \pm 0.03 & \left| \frac{f_{\Sigma^0 p K^-}}{\sqrt{4\pi}} \right| &= 0.25 \pm 0.05, \end{aligned} \quad (3.28)$$

while our results on $|f_{\Lambda p K^-}/\sqrt{4\pi}|$ are 1.35 for Set I and 0.92 for Set II, and $|f_{\Sigma^0 p K^-}/\sqrt{4\pi}|$ are 0.45 and 0.31 for Set I and II, respectively. The parameter set II seems to be better than Set I. These results are very encouraging to the model. We note that if we use the F/D ratio at $N_c = 3$, we get the $F/D = (3\sqrt{3}-1)/(3\sqrt{3}+3) = 0.507$ from $\Lambda_{\Lambda N}$ and $\Lambda_{\Sigma N}$, that is not far from $1/\sqrt{3} = 0.577$, which will be given later from the pion couplings to the hyperons. The coefficients Λ_{YN} and the pseudovector coupling constants $f_{YN\pi}/\sqrt{4\pi}$ are summarized in Table II.

Table II

3.2 Pion couplings

In order to derive the pion source term we have to calculate the second derivative of the total pion field with respect to time: $\ddot{\Phi}_a$ is given through the commutator with H_{sky} as

$$\ddot{\Phi}_a = -G^{-1}_{ab} \frac{\delta \mathcal{V}[\Phi, \nabla \Phi]}{\delta \Phi_b} + \text{terms with } \pi^2. \quad (3.29)$$

Note that if Φ_a 's are replaced by the classical fields of $O(N_c^{1/2})$, $\delta \mathcal{V}/\delta \Phi_b = 0$ is the equation of motion to the classical soliton configuration, and that the terms with π^2 are discarded, since they are of $O(N_c^{-3/2})$. Thus, the leading pion source term J_a^π comes from $(-\nabla^2 + m_\pi^2)\Phi_a$, which gives the pion coupling constants of $O(N_c^{1/2})$ to the positive parity hyperons as shown for the nucleon and Δ couplings [20]. For the positive parity hyperons the leading source term of the pion is given as

$$J_a^\pi(0) = (-\nabla^2 + m_\pi^2)\Phi_a(0), \quad (3.30)$$

and then the pion coupling constant is written as

$$\langle Y' | \widetilde{J}_a^\pi(\mathbf{q}) | Y \rangle = \langle Y' | R_{ai} \widetilde{\mathcal{J}}_i^\pi(\mathbf{q}) | Y \rangle, \quad (3.31)$$

where the rotational matrix $R_{ai} = 1/2 \text{Tr}(\tau_a \mathbf{A} \tau_i \mathbf{A}^\dagger)$ is represented by $(-1)^a D_{-a,i}^1(\Theta)$ as the function of the Euler angles of the iso-rotation, that is consistent with $A_{ai} = D_{ai}^{1/2}(\Theta)$ used previously, and

$$\widetilde{\mathcal{J}}_i^\pi(\mathbf{q}) = i q_i \frac{\omega_q^2}{q} \int d^3 r j_1(qr) f_\pi \sin F(r). \quad (3.32)$$

Then we have

$$\langle Y' | \widetilde{J}_a^\pi(\mathbf{q}) | Y \rangle = \Lambda_{Y'Y} (i \boldsymbol{\sigma} \cdot \mathbf{q}) \mathbf{I}_a^{I'I} \widetilde{G}_\pi(q) \quad (3.33)$$

for $Y' = \Sigma$ and $Y = \Sigma$ or Λ , and $(I_a^{I'I})_{I_3 I_3} = \langle I' I_3 | 1a; I I_3 \rangle$ is the transition isospin matrix from I to I' . For $Y' = Y = \Sigma$ we define the isospin matrix for $I = 1$ as $(I_a)^{I_3 I_3} = \sqrt{2} \langle 1 I_3 | 1a; 1 I_3 \rangle$. The vertex function $\widetilde{G}_\pi(q)$ is given as

$$\widetilde{G}_\pi(q) = 4\pi \frac{\omega_q^2}{q} f_\pi \int dr r^2 j_1(qr) \sin F(r). \quad (3.34)$$

For $Y' = \Sigma^*$ and $Y = \Sigma, \Lambda$,

$$\langle Y' | \widetilde{J}_a^\pi(\mathbf{q}) | Y \rangle = \Lambda_{\Sigma^*Y} (i \mathbf{S} \cdot \mathbf{q}) \mathbf{I}_a^{I'I} \widetilde{G}_\pi(q). \quad (3.35)$$

The residue of the Σ^* resonance in the elastic $\pi\Lambda \rightarrow \Sigma^* \rightarrow \pi\Lambda$ process is written as

$$(\mathbf{S} \cdot \mathbf{q}')^\dagger (\mathbf{S} \cdot \mathbf{q}) \Lambda_{\Sigma^*\Lambda}^2 \widetilde{G}_\pi^2(q) = P_3(\mathbf{q}', \mathbf{q}) \Lambda_{\Sigma^*\Lambda}^2 \frac{1}{3} \widetilde{G}_\pi^2(q), \quad (3.36)$$

$$P_3(\mathbf{q}', \mathbf{q}) = 3(\mathbf{q}' \cdot \mathbf{q}) - (\boldsymbol{\sigma} \cdot \mathbf{q}')(\boldsymbol{\sigma} \cdot \mathbf{q}), \quad (3.37)$$

where $P_3(\mathbf{q}', \mathbf{q})$ is the projection operator of $J^P = 3/2^+$. Note that an extra factor $1/3$ appears in the residue.

Setting the mass scale to the pion mass for the pion coupling constants, we define the pseudovector pion coupling constant $f_{Y'Y\pi}$ as

$$\frac{f_{Y'Y\pi}}{m_\pi} = 4\pi f_\pi \Lambda_{Y'Y} \lim_{\omega_q \rightarrow 0} \frac{\omega_q^2}{q} \int dr r^2 j_1(qr) \sin F(r), \quad (3.38)$$

because the Born term has the pole at $\omega_q = M_{Y'} - M_Y$, that is zero at leading order. The argument similar to Eq.(3.26) gives the pion coupling constant as

$$\frac{f_{Y'Y\pi}}{m_\pi} = 4\pi\Lambda_{Y'Y}f_\pi\alpha_\pi, \quad (3.39)$$

where we used the form of $F(r)$ for large r ,

$$F(r) \sim \alpha_\pi \frac{1 + m_\pi r}{r^2} e^{-m_\pi r}. \quad (3.40)$$

We find that the pion coupling constants of the hyperons are near the empirical values; $f_{\Sigma\Lambda\pi}/\sqrt{4\pi} = 0.20 \pm 0.01$ and $f_{\Sigma\Sigma\pi}/\sqrt{4\pi} = 0.21 \pm 0.02$ [24]. The pseudovector coupling constants and the coefficients are summarized in Table III, where we also give those of nonstrange nucleon and Δ . Since the ratio $f_{\Sigma+\Sigma^0\pi^+}/f_{\Sigma+\Lambda\pi^+}$ is equal to one, the F/D ratio defined at $N_c = 3$ becomes $1/\sqrt{3}$.

Table III

The coefficients give the same ratios of the pion coupling constants as the $SU(3)$ symmetry at the large N_c limit [8]; for example

$$\begin{aligned} (\Sigma^{*+} \rightarrow \Sigma^0 \pi^+) / (\Sigma^{*+} \rightarrow \Lambda \pi^+) &= -1/2 \\ (\Delta^{++} \rightarrow p \pi^+) / (\Sigma^{*+} \rightarrow \Sigma^0 \pi^+) &= -\sqrt{6}. \end{aligned}$$

The $NN\pi$ coupling constant $f_{NN\pi}$ is a little bit small for Set II than the one for Set I and the empirical value. We think, however, that the value of $f_{NN\pi}$ is much improved in Set II, since f_π is kept equal to the physical value and the axial vector coupling constant g_A becomes 1.03 in Set II compared to $f_\pi = 54$ MeV and $g_A = 0.65$ in Set I.

4 Kaon coupling to $\Lambda^*(1405)$ and negative-parity Σ states

In the bound-state approach to strangeness Λ^* is the bound state of the S-wave kaon. The S-wave bound state disappears as the kaon mass becomes small below the physical one,

for example the zero-energy bound state appears near $m_K \sim 300\text{MeV}$ for the parameter set II, that is, the bound-state pole on the physical sheet moves to a resonance pole on the unphysical sheet of the $\bar{K} - N$ scattering amplitude.

The kaon wave function in the intrinsic frame is given as

$$\mathcal{K}_S(\mathbf{r}, t) = k_0^*(r) e^{i\omega_0 t} \begin{pmatrix} a_{-1/2}^\dagger Y_{00}^* \\ -a_{1/2}^\dagger Y_{00}^* \end{pmatrix} \equiv k_0^\dagger(r) e^{i\omega_0 t} \boldsymbol{\Omega}_0(a^\dagger; \theta, \phi), \quad (4.1)$$

where k_0 is the radial wave function, and $a_t^\dagger = a_{0,1/2,t}^\dagger$. The Λ^* state is expressed as

$$\begin{aligned} |\Lambda^*; j_3 > &= \sum_t \sqrt{\frac{1}{8\pi^2}} D_{0, j_3-t}^0(\Theta) a_t^\dagger |0 > \\ &= \sqrt{\frac{1}{8\pi^2}} D_{0,0}^0(\Theta) a_{j_3}^\dagger |0 >. \end{aligned} \quad (4.2)$$

Then, we have

$$< \Lambda^*; J_3 | A_{\alpha i} \widetilde{\mathcal{J}}_i^K(k) | N; j_3 > = \frac{1}{\sqrt{2}} \widetilde{G}_0(k) \delta_{J_3 j_3}, \quad (4.3)$$

where

$$\widetilde{G}_0(k) = \sqrt{4\pi} (\omega_k^2 - \omega_0^2) \int dr r^2 j_0(kr) k_0(r) \quad (4.4)$$

with ω_0 being the S-wave bound-state energy. The pseudoscalar kaon coupling constant to $K\Lambda^*N$ is of $\mathcal{O}(N_c^0)$ as the same as the kaon coupling constants for the positive parity hyperons and given as follows: Since the pole is at $\omega_k = M_{\Lambda^*} - M_N = \omega_0 + \mathcal{O}(N_c^{-1})$, we have

$$\begin{aligned} G_{\Lambda^*NK} &= \sqrt{4\pi} \Lambda_{\Lambda N} \lim_{\omega_k \rightarrow \omega_0} (\omega_k^2 - \omega_0^2) \int dr r^2 j_0(kr) k_0(r) \\ &= \sqrt{4\pi} \Lambda_{\Lambda N} \alpha_0 \kappa_0, \end{aligned} \quad (4.5)$$

where $\kappa_0 = \sqrt{m_K^2 - \omega_0^2}$ and asymptotically

$$k_0(r) \sim \frac{\alpha_0 \kappa}{r} e^{-\kappa_0 r}. \quad (4.6)$$

Note that $\alpha_0 \kappa_0$ is dimensionless.

We get

$$G_{\Lambda^*pK^-}/\sqrt{4\pi} = -1.45 \quad \text{and} \quad -0.72 \quad (4.7)$$

for Set I and II, respectively. According to the phenomenological analysis of the Λ^* resonance contribution to KN and $\overline{K}N$ scattering length gives it to be 0.75 and 0.58 for $\bar{g}_{\Lambda_R}^2 = 0.25$ and 0.15, respectively, where $|G_{\Lambda^*pK^-}| = \bar{g}_{\Lambda_R} m_K / f_\pi$ [26]. So, the coupling constant in Set I seems to be too large. However, the experimental value of $\text{Im}A_{K^-p} \sim 0.7$ fm would restrict the upper bound of $G_{\Lambda^*pK^-}^2/\sqrt{4\pi}$ to about 0.6 for $\Gamma^* = 50$ MeV with Γ^* being the decay width of Λ^* , because the imaginary parts coming from various channels sum up due to unitarity of the elastic amplitude and the imaginary part of the resonance amplitude is written as

$$i \frac{\Gamma^*/2}{(M_{\Lambda^*} - M_N - m_K)^2 + (\Gamma^*/2)^2} G_{\Lambda^*pK^-}^2. \quad (4.8)$$

Since the $SU(2)$ soliton has the rotational $I = J$ band with I being an integral number 0, 1, \dots for $S = -1$ channel, the bound-state approach generates Σ states with negative parity besides the Λ^* state. We denote these states as $\Sigma_{1/2}^-$ and $\Sigma_{3/2}^-$ with spin 1/2 and 3/2, respectively. Here we examine whether that Σ_S^- can interact with non-exotic channels such as $\overline{K}N$ and πY channels.

The $\Sigma_{1/2}^-$ state is written as

$$|\Sigma_{1/2}^-; I_3, J_3 \rangle = \sqrt{\frac{3}{8\pi^2}} \sum_{t=\pm 1/2} (-1)^{1+I_3} \langle \frac{1}{2}, J_3 | 1, J_3 - t; \frac{1}{2}, t \rangle D_{-I_3, J_3-t}^1 a_t^\dagger |0 \rangle. \quad (4.9)$$

The $\Sigma_{1/2}^- N \overline{K}$ coupling constant is given as

$$\langle \Sigma_{1/2}^-; J_3, I_3 | A_{\alpha i} \widetilde{\mathcal{J}}_i^K(k) | N; j_3 \rangle = \Lambda_{\Sigma_{1/2}^- N} \widetilde{G}_0(k) \delta_{J_3 j_3}, \quad (4.10)$$

where $\Lambda_{\Sigma_{1/2}^0 p} = 1/\sqrt{2}$ and $\Lambda_{\Sigma_{1/2}^+ p} = -1$. $\Sigma_{1/2}^-$ would couple to $\Lambda\pi$ and $\Sigma\pi$ as like as $G_{\Lambda^*\Sigma\pi}$, which are of $O(N_c^{-1/2})$. Thus, the $\Sigma_{1/2}^-$ state can strongly interact with $\overline{K}^0 p$ and $\pi\Sigma$, which are the elastic channels.

As to $\Sigma_{3/2}^-$ the model predicts that it cannot interact with the elastic $\overline{K}N$ channel, but can do with the $\overline{K}\Delta$ at $O(N_c^0)$:

$$\langle Z_{3/2}^-; I_3, J_3 | A_{\alpha i} \widetilde{\mathcal{J}}_i^K(k) | \Delta; i_3, j_3 \rangle = \Lambda_{\Sigma\Delta} \widetilde{G}_0(k) \delta_{J_3 j_3}. \quad (4.11)$$

The pion couplings among the negative-parity states Λ^* , $\Sigma_{1/2}^-$ and $\Sigma_{3/2}^-$ occur at leading order $O(N_c^{1/2})$; for example,

$$\langle \Sigma_{3/2}^-; I_3, J_3 | J_a^\pi(q) | \Lambda^*; j_3 \rangle = -\frac{1}{\sqrt{3}} i(\mathbf{S} \cdot \mathbf{q}) \tilde{G}_\pi(q), \quad (4.12)$$

but it is also impossible to couple to the πY channels in the model. Thus, the $\Sigma_{3/2}^-$ could not interact with the elastic channels.

Is there a candidate for the $\Sigma_{1/2,1/3}^-$ state? If c_0 is larger than c_1 , the mass difference between $\Sigma_{1/2}^-$ and Λ^* cannot be larger than the mass difference between Σ and Λ . Therefore, $\Sigma_{1/2}^-$ could not be attributed to the established $\Sigma(1750)$, because the mass spacing from Λ^* is too large. There is an indication of an enhancement near 1480 MeV in the $\overline{K}^0 p$ mass spectrum, whose spin and parity are not known [29,30]. $\Sigma_{1/2}^-$ may be attributed to this $\Sigma(1480)$, though it is not yet established, and if $\Sigma_{1/2}^-$ exists really, it would lie above but not far from the $\overline{K}N$ threshold. The resonance above the $\overline{K}N$ threshold contributes a positive value to the K^-n scattering length. The mass difference between $\Sigma_{3/2}^-$ and Λ^* would be larger than $c_0(M_\Delta - M_N)$, but since it does not interact with the elastic channels, it may be difficult to observe the $\Sigma_{3/2}^-$ state.

5 Conclusions and discussion

We have formulated the pion and kaon coupling constants to baryons with strangeness within the bound-state approach to strangeness in the Skyrme soliton model. The positive parity hyperon $\Lambda(1115)$, $\Sigma(1192)$ and $\Sigma^*(1385)$ appear as the bound states of the P-wave kaon to the $SU(2)$ soliton, whereas the $\Lambda^*(1405)$ does as the S-wave bound state in this approach.

The pion fields used in the Lagrangian and Hamiltonian are defined as the total fields consisting of the classical Skyrmon fields and the fluctuation. The kaon fields are introduced as the fluctuation around the $SU(2)$ soliton in the laboratory system according to the Callan-Klebanov ansatz [3]. The meson-baryon vertices are defined as the source terms of the pion and kaon fields sandwiched between the single-baryon states. The sandwiched source term is rewritten in terms of the variables in the intrinsic frame of the soliton; the pion field becomes the Skyrmon field and the kaon field does the bound-state one. The kaon coupling to the positive-parity hyperon is of nonrelativistic pseudovector type. The coupling constant is defined as the residue at the pole of the Born term. According to this definition the magnitude of the kaon coupling constant is controlled by the asymptotic behavior of the normalized bound-state wave function as like as the pion coupling constant controlled by the asymptotic behavior of the chiral angle. Thus, if the equation to the bound state is the same, irrespectively to the ansatz by Callan-Klebanov or Blom et al, the resultant coupling constant does not depends on the ansatz adopted.

The order of the coupling constants in the $1/N_c$ expansion is such that f_{YNK} is of $O(N_c^0)$, $f_{Y'Y\pi}$ of $O(N_c^{1/2})$ and G_{Λ^*NK} is of $O(N_c^0)$. We found that the kaon vertex to the hyperon is of nonrelativistic pseudovector type as the same as the πNN vertex. Since the Λ^* state is the bound state of the S-wave kaon, strangeness cannot flows within the baryon line from Λ^* to the positive-parity hyperon states. In order to obtain $G_{\Lambda^*\Sigma\pi}$, we have to construct higher order pion source terms of $O(N_c^{-1/2})$, which are bilinear in $\bar{K}K$. This elaborate task to construct the higher order source term of the pion will appear elsewhere.

Our meson-baryon vertices are much more simple and transparent than those given in Refs.[18,19] with respect to the definition of the vertices. If we adopt the $SU(3)$ collective-coordinate method instead of the bound-state approach, the Lagrangian for the kaon fields as the fluctuation around the nonstrange soliton is written as bilinear forms of

the kaon fields as the same as the bound-state approach. But it cannot straightforward give the Yukawa vertices, because the hyperons appear as the $SU(3)$ rotating soliton and do not involve any kaons at tree level as in the $SU(2)$ case for the pion vertices. The Yukawa coupling of the kaon occurs as higher order term coming from the fact that the rotating solitons breaks the equations of motion. Our method can be applicable to the $SU(3)$ collective-coordinate method as in the $SU(2)$ case [31], though it would be much involved.

From the comparison of the calculated kaon and pion coupling constants with the phenomenological analyses [24,25, 26], we found that the parameters of the model in Set II are better than those in Set I. We also note that the magnitude of g_A of the nucleon is $g_A = 1.03$ in Set II, but it is 0.65 in Set I. Although the value of f_π is fixed to the empirical value 93 MeV, the Skyrme constant is set to a rather small value, $e = 4.0$ in Set II. Such a small value of the Skyrme constant could not be supported by the chiral perturbation theory in the meson sector, but it seems that the Skyrme Lagrangian with the Set II parameters, having no chiral six order terms, is effectively equivalent to the one with a large value of the Skyrme constant supplemented by the standard six order term $-1/2e_6^2 B^\mu B_\mu$ [22,23]. The sizes of the kaon coupling constants in Set I in our method are about twice as large as those of Refs. [18, 19].

Since the $SU(2)$ soliton has the rotational $I = J$ band with I being an integral number 0, 1, \dots for $S = -1$ channel, the bound-state approach generates baryons with the isospin such as $I \geq 2$ for the P-wave bound state and $I \geq 1$ for the S-wave bound state. All of the Σ states with negative parity are not exotic, because the $\Sigma_{1/2}^-$ can interact with the $\bar{K}N$ and πY channels as discussed in the previous section. If the low-lying Σ states with negative parity predicted by the model are not observed at all, the prediction of the Σ states with negative parity may be a defect of the bound-state approach to Skyrme model. It is important, therefore, to reveal experimentally low energy resonances in the

$\overline{K}N$ and πY channels for the validity of the model. Nevertheless, we emphasize that the model can be applicable to low energy physics in great variety as an effective theory.

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Appendix Hamiltonian in the intrinsic frame

If we sandwich the total Hamiltonian $H = \int d^3x \mathcal{H}$ between two single-hyperon states $|Y(\mathbf{p})\rangle$,

$$\langle Y(\mathbf{p}') | H[\Phi, \pi; K, \Pi] | Y(\mathbf{p}) \rangle,$$

the total field Φ_a and π_a are reduced to the classical Skymion configuration in the tree approximation [20], and also the kaon fields to the bound-state fields with the specific angular momentum, both of which are defined in the intrinsic frame of the Skyrme soliton.

These are defined in Eqs.(3.9) to (3.13). The time-derivatives of them are given as

$$\dot{K}(x) = \mathbf{A}(t)\dot{\mathcal{K}} + \dot{\mathbf{A}}\mathcal{K} - \dot{\mathbf{X}} \cdot \nabla \mathcal{K}, \quad (\text{A.1})$$

$$\dot{\Phi}_a(x) = \dot{R}_{ai}\hat{\phi}_i - R_{ai}\dot{\mathbf{X}} \cdot \nabla \hat{\phi}_i \quad (\text{A.2})$$

with

$$\mathbf{A}^\dagger \dot{\mathbf{A}} = \frac{i\tau_a}{2} \dot{\Theta}_a, \quad (\text{A.3})$$

$$R_{aj}\dot{R}_{ai} = \varepsilon_{jib}\dot{\Theta}_b,$$

where $\dot{\Theta}_a$ is the angular velocity around the a-th iso-spin axis and \mathbf{X} is the center of the Skymion. Since the mixing terms of the rotational and translational modes vanish or are of higher order, $\dot{\Theta}_a$ and \dot{X}_i are given separately by their conjugate momenta, which are defined as

$$I_a = \frac{\partial L}{\partial \dot{\Theta}_a} = \Lambda_s \dot{\Theta}_a - c_\ell T_a, \quad (\text{A.4})$$

$$P_i = \frac{\partial L}{\partial \dot{X}_i} = M_s \dot{X}_i + P_i^K, \quad (\text{A.5})$$

where I_a is the angular momentum of the soliton in the intrinsic frame, \mathbf{T} is the spin of the bound kaon, $(T_a)_{tt'} = a_t^\dagger(\tau_a/2)a_{t'}$, and the constant c_ℓ depends on the angular momentum of the bound kaon, which are given as

$$c_1 = 1 - \int dr r^2 k_1^2 \omega_1 \left\{ \frac{4}{3}(1+c)\left(f + \frac{s^2}{2\kappa^2 r^2}\right) - \frac{1}{\kappa^2 r^2} \frac{d}{dr} (r^2 F' s) \right\}, \quad (\text{A.6})$$

$$c_0 = 1 - \int dr r^2 k_0^2 \omega_0 \left\{ \frac{4}{3}(1-c)\left(f + \frac{s^2}{2\kappa^2 r^2}\right) + \frac{1}{\kappa^2 r^2} \frac{d}{dr} (r^2 F' s) \right\}. \quad (\text{A.7})$$

For the linear momentum \mathbf{P}^K denotes the kaon momentum defined as

$$\mathbf{P}^K = - \int d^3 r \{ \hat{\Pi}^\dagger \nabla \mathcal{K} + h.c. \}, \quad (\text{A.8})$$

where $\hat{\Pi}$ is the momentum field in the intrinsic frame, which can be defined as $f\dot{K}_\beta + i\lambda K_\beta$. Thus, we can regard $\mathbf{P}_S = \mathbf{P} - \mathbf{P}^K$ as the baryon momentum. We note that there are additional higher order terms in Eq.(A.5), which vanish for the bound-state kaon, because the bound-state kaon has a definite angular momentum and parity.

Then we have the intrinsic Hamiltonian responsible to the bound-states in the tree approximation:

$$H = M_S + \sum_{\ell=0,1} \frac{(\mathbf{I} + c_\ell \mathbf{T})^2}{2\Lambda_S} + \frac{\mathbf{P}_S^2}{2M_S} + \sum_{\ell=0,1, t=\pm 1/2} \omega_\ell a_{\ell,t}^\dagger a_{\ell,t}. \quad (\text{A.9})$$

We see that $\mathcal{H}_{\pi K}$ is absorbed into the first term through the transformation from the laboratory to the intrinsic frame. Thus, we see that

$$\begin{aligned} < Y(\mathbf{p}') | H[\Phi, \pi; K, \Pi] | Y(\mathbf{p}) > = \left(M_s + \omega_\ell + \frac{(\mathbf{I} + c\mathbf{T})^2}{2\Lambda_S} + \frac{\mathbf{p}^2}{2M_s} \right) \delta(\mathbf{p}' - \mathbf{p}) \\ &= \left(M_Y + \frac{\mathbf{p}^2}{2M_Y} \right) \delta(\mathbf{p}' - \mathbf{p}) + \mathcal{O}(N_c^{-2}), \end{aligned} \quad (\text{A.10})$$

which is the nonrelativistic energy of the hyperon in the tree approximation.

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Tables and Table Captions

Table I Bound-state parameters. The parameters of the model are taken to be $f_\pi = 54$ MeV and $e = 4.84$ in Set I, and $f_\pi = 93$ MeV and $e = 4.0$ in Set II. The pion and kaon masses are taken as 138 MeV and 495 MeV, respectively, for the both sets.

Table II Λ_{YN} and the pseudovector coupling constants f_{YNK} . The pseudoscalar coupling constant is given by $G_{YNK} = (M_Y + M_N)/m_K \cdot f_{YNK}$. The empirical values with *) are taken from ref. 24), and those with **) from ref. 25).

Table III $\Lambda_{Y'Y}$ and the pseudovector coupling constants. $f_{NN\pi}$ and $f_{\Delta N\pi}$ are also given for comparison. $G_{Y'Y\pi} = (M_Y + M_{Y'})/m_\pi \cdot f_{Y'Y\pi}$. The empirical values with *) are taken from ref. 24), where the empirical value $f_{\Sigma^*\Lambda\pi}$ is multiplied by the extra factor $\sqrt{3}$. (See Eq.(3.36).)

	$\omega_1(\text{MeV})$	$\omega_0(\text{MeV})$	c_1	c_0	$1/\Lambda_S(\text{MeV})$
Set I	147	339	0.513	0.816	195
Set II	183	434	0.388	0.788	167

Table I

	Λ_{YN}	$ f_{YNK}/\sqrt{4\pi} $		
		Set I	Set II	empirical
$\Lambda_{\Lambda p} = -\Lambda_{\Lambda n}$	$1/\sqrt{2}$	1.35	0.92	$0.89 \pm 0.10^{*)}$ $0.94 \pm 0.03^{**})$
$\Lambda_{\Sigma^+ p} = \Lambda_{\Sigma^- n}$	$-1/3$	0.64	0.43	
$\Lambda_{\Sigma^0 p} = \Lambda_{\Sigma^0 n}$	$-1/3\sqrt{2}$	0.45	0.31	$< 0.43 \pm 0.07^{*)}$ $0.25 \pm 0.05^{**})$
$\Lambda_{\Sigma^{*+} p} = \Lambda_{\Sigma^{*-} n}$	$-2/\sqrt{3}$	2.21	1.50	
$\Lambda_{\Sigma^{*0} p} = \Lambda_{\Sigma^{*0} n}$	$-\sqrt{2/3}$	1.55	1.06	

Table II

$Y'Y$	$\Lambda_{Y'Y}$	$ f_{Y'Y\pi}/\sqrt{4\pi} $		
		Set I	Set II	empirical
$\Sigma\Lambda$	$1/3$	0.25	0.22	$0.20 \pm 0.01^{*)}$
$\Sigma\Sigma$	$1/3$	0.25	0.22	$0.21 \pm 0.02^{*)}$
$\Sigma^*\Lambda$	$-1/\sqrt{3}$	0.43	0.38	$0.25 \pm 0.01^{*)}$
$\Sigma^*\Sigma$	$1/\sqrt{12}$	0.21	0.19	
NN	$-1/3$	0.25	0.22	0.27
ΔN	$-1/\sqrt{2}$	0.54	0.47	0.47

Table III